# Chapter Six – Network Problems Walkthru

This chapter utilizes the same basic structures for linear programming optimization problems, but just applies them to new types of problems. In the chapter six case, network and transportation problems.

In this chapter we are looking at flows of things through a network. Each network has two basic things:

* ***Nodes*** – which are points where what is flowing through the network can take different paths. These can be warehouses or the center of a small city that you must visit and then leave.
* ***Arcs*** – these are the paths where the flow travels.

Commonly there are three types of nodes:

* ***Supply nodes*** – this is where the material that will flow through the network originates. Each supply node usually has a limited supply that must be passed through the network.
* ***Demand nodes*** – this are the nodes at the end of the network that will receive the supply that is created by the supply nodes.
* ***Transshipment nodes*** – these are the nodes where the flow of the material can take different paths. I tend to think of them as forks in the road.

For the arcs, there are two things to consider:

* Arcs are paths that can be taken. They (like a road) are each a certain distance (or cost or time) if they are taken. They can each be different lengths (or costs, or time).
* Each arc may also have a limit to the amount of flow that can be send down the arc – a capacity. Some models have limited capacity arcs, some do not.

The key concept to understanding network diagrams is in the ‘net flow’ concept.

## The Spreadsheet

Open the book to page 194 and also open the Distribution Unlimited spreadsheet from the CD, or from my web site ([www.karlknapp.com/academics/mba620/Distribution](http://www.karlknapp.com/academics/mba620/Distribution) Unlimited.xls).

### Arcs

On the left side of the sheet you will see each arc in the network. Each arc is listed in a row labeled with the beginning (From) and ending (To) nodes that lay between the two end points of the arc. Each and every arc must be listed in the rows at the left side of the spreadsheet.

If you look at Figure 6.2 on the top of page 191, you will see that the arcs lists match the drawn network diagram.

The ***decision variables*** in most network problems are how much material should flow along which arcs to minimize (usually cost, time or distance). In this case it is to minimize cost. The Ship column contains the decision variables of how much material should flow along each arc.

### Arc Capacities

Some, but not all of the arcs have ***limited capacity***. If you notice in the diagram on page 191, the arcs at the very top and very bottom DO NOT have capacities listed. Since they do not, they are not limited.

The other arcs all list a certain maximum capacity. These capacities are listed in column F of the spreadsheet. As you can guess, the <= sign is a dead giveaway for a linear programming constraint. The amount shipped along an arc (column D) cannot exceed the capacity of that arc (column F).

### Arc Costs

In this problem we are attempting to minimize total shipping cost. Each arc traveled has a unit cost associated with it for each item that uses that arc. The unit costs for each arc are listed in the diagram on page 191. You can see these unit costs also listed in column G of the spreadsheet.

### Objective Function – Minimize Total Cost (Time or Distance)

If you look at the formula in cell D11, you will see something familiar, our old friend the SumProduct formula. In this case we are attempting to minimize total cost. The total cost is the number of items shipped along each arc multiplied by the unit cost to ship an item along that same arc. The objective function takes each decision variable cell and multiplies it by the unit cost in column G, and then adds them all up (sumproduct).

### Nodes

Ok, we have covered all of the arcs in our network. Now we move on to the nodes. All of the nodes in the problem are listed on the right hand side of the spreadsheet in column I. Their labels MUST MATCH the labels used to describe the arcs EXACTLY.

### Net Flow

**Net flow is the key concept in this chapter**. The Net Flow is calculated by the following formula:

Total Flow From The Node – Total Flow To The Node

#### Supply Nodes

***Supply nodes*** are the points where the flow begins. They provide flow to the network but do not receive any. In this case, the two factories F1 and F2 are the supply nodes, each supplying 80 and 70 units respectively.

The ***NET FLOW FROM ALL SUPPLY NODES IS POSITIVE***. The net flow from node F1 is 80 because it is supplying 80 (total flow from the node) and receiving none (total flow to the node).

#### Demand Nodes

***Demand nodes*** are the points where the flow ends. They do not provide any flow to the network, but receive the flow. In this case, the two warehouses W1 and W2 are the demand nodes, each needing to receive 60 and 90 units respectively.

The ***NET FLOW FROM ALL DEMAND NODES IS NEGATIVE***. The net flow from node W1 is -60 because it is supplying 0 (total flow from the node) and receiving 60 (total flow to the node).

#### Conservation of Flow

Notice that the total amount supplied is the same as the total amount demanded. This is called the ***conservation of flow***.

#### Transshipment Nodes

In this case the distribution centers (DCs) are ***transshipment nodes***. They may (or may not) receive flow from the factories, but if they do they do not store any of the items. Everything received at a transshipment node must be passed through the network to the demand nodes.

The ***NET FLOW AT ALL TRANSSHIPMENT NODES IS ZERO***. The net flow at any transshipment node must be zero because everything received (total flow to the node) must be shipped (total flow from the node). It is the zero net flow that ensures that no items end up stranded at a transshipment node.

### Net Flow Calculation in the Spreadsheet

The book examples utilize a nifty function called **SUMIF**. This function sums the values of cells only if a condition specified is met.

Look at the formula in cell J4:

*=SUMIF(From,I4,Ship)-SUMIF(To,I4,Ship)*

This is the key formula in the spreadsheet. The first SUMIF represents the total flow from the node. The second sumif represents the total flow to the node.

Remember, net flow = total flow from the node – total flow to the node.

Lets look at the first SUMIF. It uses two named ranges. The From named range contains cells B4 to B9. The Ship named range are the decision variables in cells D4 to D9. This function looks at the value of cell I4 (which contains the value F1). It looks at all of the cells in the named range From, and IF it finds a match (only in rows 4 and 5) it sums the values in the corresponding row in the Ship range. In the case of Cell J4, it finds matches to the value F1 in cells B4 and B5. Since it found matches, it sums the amounts listed in the From range, 30 (in row 4) and 50 (in row 5).

The second SUMIF works the same way. The only difference is that it is looking to match the value in cell I4 to the receiving node listed in column D. In the case of Cell J4, F1 does not receive any flow because there are no arcs that terminate in node F1. Since it finds no matches, the SUMIF returns 0.

The net flow is total flow from the node (80) – total flow to the node (0) = 80 (supply node).

Each node in the network must be listed on the right side of the spreadsheet with its corresponding supply/demand listed in column L.

### Net Flow Constraints

If you open Solver, you will find only two constraints. They are:

* the capacities of each arc, and
* the net flow at each node

### RUN Solver

Clear the decision variables in column D. Run solver and review any section of this description until you are sure you understand the model.

## PRACTICE

Utilize the Distribution Unlimited template to complete problem 6.3 on page 216.

1. Draw the network diagram just like the one on page 191.
2. Then modify the spreadsheet with the proper arcs, nodes, capacities, unit costs and supply/demands (net flows).
3. Run the spreadsheet.
4. To check your answers, you should have gotten a total cost of $488,125.

# MAXIMUM FLOW PROBLEMS

A similar type problem is a ***maximum flow problem***. It has the same layout as before, but without unit costs. The only difference is that the objective function is to maximize the total flow.

Open the BMZ spreadsheet example. Review the objective function in cell D14.

This first problem only has one supply and one demand node. The Expanded BMZ problem has multiple. Same formulation.

Open the Expanded BMZ spreadsheet and review the objective function in cell D21. It is the total flow into both demand nodes.

# SHORTEST PATH PROBLEMS

The last type of problem I want you to focus on is the ***shortest path problem*** on page 204.

Here we are trying to find the shortest path for the fire department in a town. Open the Littletown spreadsheet from the CD or my web site.

The network is the same layout in the spreadsheet. Arcs on the left, nodes on the right. Each arc has a distance this time instead of a cost (in the real world you can even use maps.google.com to find the distance or time between each of your facilities – the time may be different just by going in the opposite direction. In this network notice that the arcs do not have arrows. The flow may move in either direction along an arc. This is represented by TWO ARCS in the spreadsheet. Notice that the arc B to C (row 10) and C to B (row 13).

The nodes are again on the right side.

To force a shortest path solution, the only difference is that there is one supply node, supplying ONLY 1 unit and one demand node demanding only 1 unit.

When you run Solver the 1 is shown in the decision variables if that arc is on the path – and 1 unit is flowing along the arc.

# HOMEWORK

Utilize this write up and the templates to do homework problems 6.2, 6.4, 6.5 and 6.6.